Group – D

6. (a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$, if it exists.

(b) Assuming the convergence of the integral prove that, $\int_0^{\infty} \sqrt{x}\, e^{-x^3}\, dx = \frac{\sqrt{\pi}}{3}$.

6 + 6 = 12

7. (a) Evaluate: $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$

(b) Solve by Laplace Transform:
$y''(t) + y(t) = \sin 2t$, where $y(0) = y'(0) = 1$.

6 + 6 = 12

Group – E

8. (a) Show that the straight lines whose direction cosines are given by $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$ are parallel if $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$.

(b) A variable plane at a constant distance p from the origin 0 meets the axes at A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$.

6 + 6 = 12

9. (a) Find the equations of the lines of greatest slope and least slope on the plane $3x-4y+5z-5=0$ drawn through the point (1, 2, 2), given that the plane $4x-5y+6z-6=0$ is horizontal.

(b) Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1, x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if *2d* is the shortest distance, prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$.

6 + 6 = 12

B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/2ND SEM/MATH 1201/2017

MATHEMATICS - II
(MATH 1201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) $\frac{1}{D^2+1}\sin 2x =$

(a) $\frac{1}{3}\sin 2x$ (b) $\frac{1}{3}\cos 2x$ (c) $-\frac{1}{3}\sin 2x$ (d) $\frac{1}{5}\sin 2x$

(ii) The general solution of $\frac{d^2y}{dx^2}=0$ is

(a) $y=a$ (b) $y=ax+b$
(c) $y=ax^2+bx+c$ (d) $y=a\cos x+b\sin x$.
(where a, b, c are arbitrary constants).

(iii) Integrating factor of $x\frac{dy}{dx}+y=xe^x$ is

(a) x (b) e^x (c) 1 (d) e^{-x}.

(iv) The minimum number of edges in a connected graph having 21 vertices is

(a) 18 (b) 20 (c) 10 (d) 11.

(v) The number of pendant vertices in a binary tree with n vertices is
(a) n-1 (b) n (c) n+1 (d) (n+1)/2.

(vi) $L\{e^{-t}\sin 2t\}=$

(a) $\frac{1}{s^2+2s+5}$ (b) $\frac{2}{s^2+2s+5}$
(c) $\frac{s}{s^2+2s+5}$ (d) $\frac{s+1}{s^2+2s+5}$.

(vii) $L^{-1}\left\{\dfrac{1}{(p+1)(p+3)}\right\} =$

(a) $\frac{1}{2}\left(e^{t}-e^{3t}\right)$ (b) $\left(e^{-t}-e^{-3t}\right)$ (c) $\frac{1}{2}\left(e^{-t}-e^{-3t}\right)$ (d) $\frac{1}{2}\left(e^{-3t}-e^{-t}\right)$.

(viii) $\Gamma\left(\frac{7}{2}\right)$ is equal to

(a) $\frac{5\sqrt{\pi}}{8}$ (b) $\frac{3\sqrt{\pi}}{4}$ (c) $\frac{15\sqrt{\pi}}{8}$ (d) $\frac{\sqrt{\pi}}{2}$.

(ix) A directed line makes angles 60^o and 45^o with the axes of x and y respectively. What angle does it make with the axis of z?

(a) 60^o (b) 45^o (c) 30^o (d) 90^o.

(x) The length of the perpendicular from the origin upon the plane $2x+6y-3z+5=0$ is

(a) 5/7 (b) 6/7 (c) 3/7 (d) 5.

Group – B

2. (a) The equation $\dfrac{x^2}{a^2+\lambda}+\dfrac{y^2}{b^2+\lambda}=1$ (where a and b are fixed constants and λ is an arbitrary parameter which can assume all real values) represents a family of confocal conics. Obtain the differential equation of this family.

(b) Determine the exactness of the following differential equation:

$$\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\left(x+\log x-x\sin y\right)dy=0$$

(c) Solve: $e^y-p^3-p=0$

4 + 2 + 6 = 12

3. (a) Solve: $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+4y=x\sin(\log x)$

(b) Solve the following differential equation using the method of variation of parameters:

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$

6 + 6 = 12

B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/2ND SEM/MATH 1201/2017

Group – C

4. (a) What will be the least possible number of simple regular graphs having 20 edges? Justify your answer.

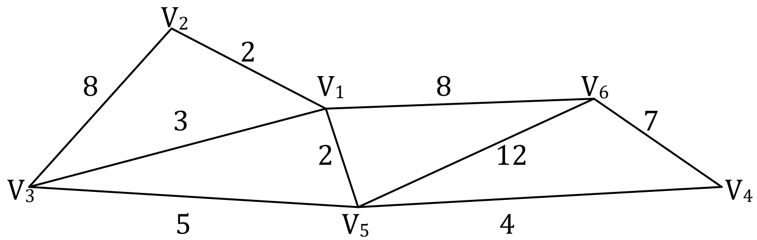
(b) Draw the digraph with the following adjacency matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

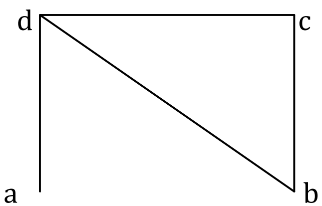
(c) Let G be a graph with 15 vertices and 4 components. Prove that, G has at least one component having at least 4 vertices.

5 + 4 + 3 = 12

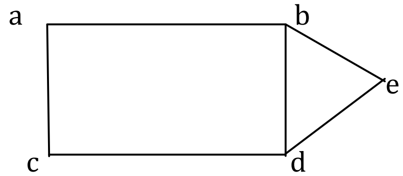
5. (a) Use Kruskal's algorithm to find a minimal spanning tree for the following graph:



(b) Find and hence draw all possible spanning trees in the following graph:



(c) Using BFS and DFS algorithms, find spanning trees of the following graph:



4 + 3 + 5 = 12